

An Infinite Charge Plane

Say that we have a **very large** charge disk. So large, in fact, that its radius a approaches **infinity** !

Q: *What electric field is created by this infinite plane?*

A: We **already** know! Just evaluate the charge disk solution for the case where the disk **radius** a is **infinity**.

In other words:

$$\lim_{a \rightarrow \infty} \mathbf{E}(x=0, y=0, z) = \begin{cases} \hat{a}_z \frac{\rho_s}{2\epsilon_0} \left[1 - \frac{z}{\sqrt{z^2 + a^2}} \right] & \text{if } z > 0 \\ \hat{a}_z \frac{\rho_s}{2\epsilon_0} \left[-1 - \frac{z}{\sqrt{z^2 + a^2}} \right] & \text{if } z < 0 \end{cases}$$

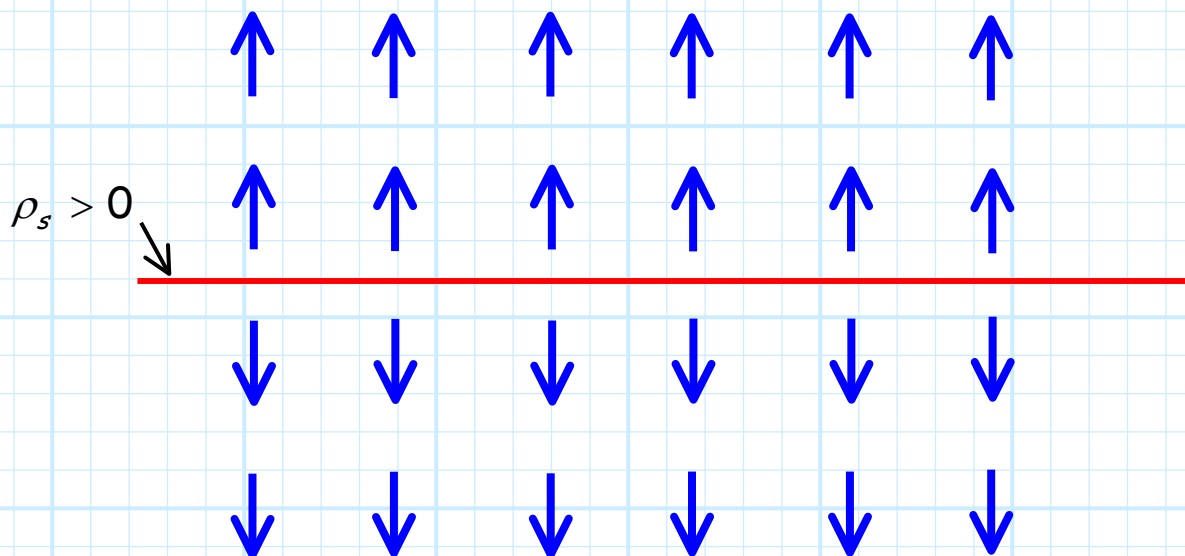
$$= \begin{cases} \frac{\rho_s}{2\epsilon_0} \hat{a}_z & \text{if } z > 0 \\ -\frac{\rho_s}{2\epsilon_0} \hat{a}_z & \text{if } z < 0 \end{cases}$$

Therefore, the electric field produced by an **infinite charge plane**, with surface charge density ρ_s , is:

$$\mathbf{E}(\vec{r}) = \begin{cases} \frac{\rho_s}{2\epsilon_0} \hat{a}_z & \text{if } z > 0 \\ \frac{-\rho_s}{2\epsilon_0} \hat{a}_z & \text{if } z < 0 \end{cases}$$

Think about what **this** says!

- * First, we note that the electric field **points away** from the plane if ρ_s is positive, and toward the plane if ρ_s is negative.
- * Second, we notice that the magnitude of the electric field is a **constant**—the magnitude is **independent** of the distance from the infinite plane!



The reason for this result is, that no matter how far you are (i.e., $|z|$) from the infinite charge plane, you remain **infinitely close** to plane, when **compared** to its radius a .

We will find these results are useful when we study the behavior of a parallel plate **capacitor**.